Conventions in Retroanalysis

A Plea for Consistency and Completeness

Alexander George

Conventions in Retroanalysis:
A Plea for Consistency
and Completeness

Alexander George

Amherst, Massachusetts

2002

Retrograde analysis, or retroanalysis for short, consists in the application of logic to determine past play on the basis of information about a given board position and the rules of chess. Composers have shown much ingenuity in thinking up questions that can be answered only on the basis of information about past play (*e.g.*, "Which Bishop is promoted?," "What was the Rook's path?," "Where is the Black King standing?") and in arranging the pieces so as to force their answers. Even the general problemist, who is less keen on composing or solving retroanalytic chess problems, must take an interest in them: for a longstanding constraint on chess problems of all kinds is that their initial position be a legal one, that is, one obtainable by a sequence of legal moves from the initial game position. It is striking that despite this, and despite more than a century of intensive retroanalytic work, there is still unclarity and disagreement concerning basic principles. It is the aim of this essay to indicate some sources of discontent, to assess various responses, and, finally, to advance and defend a view that, while not new, is deserving of a more precise articulation and a broader support than it has hitherto received.

Retroanalysis in chess is enriched by the existence of moves whose legality depends on features of the history of the game position. There are only two such moves in chess: *en passant* (*e.p.*) capture and castling. They are in a certain respect dual to one another: *e.p.* capture is legal only if a certain move *has* been made, while castling is legal only if certain moves *have not* been made.

Because *e.p.* capture requires the existence of a particular previous move (by the captured pawn), it is natural to adopt as a convention that it shall be permitted when and only when it can be established that the required move has taken place. And indeed this is the convention that has been universally adopted.

It is tempting to extend this convention to castling and declare that it shall be permitted when and only when it can be established that the designated moves (by the king or the relevant rook) have not taken place. This would have the effect, however, of ruling out all castling, since it is impossible to prove that such moves have never taken place. The search is thus encouraged for a convention that makes reference to the existence of certain moves, rather than to their non-existence. And this leads, naturally enough, to the common convention for castling (CCC), which declares it legal unless it can be proved not to be, that is, unless it can be established that one of the designated moves has occurred. We might put this by saying that, while the convention regarding *e.p.* capture places the burden of proof on those who claim it is legal, the convention for castling places it on those who hold it to be illegal.

The CCC is widely assumed in the construction of retroanalytical chess problems.[1] It was even officially adopted in 1958 at the International Congress of Problemists at Piran, Yugoslavia. What has come to be known as the Piran Codex asserts that: "CASTLING is always regarded as legal whenever its illegality cannot be proved."[2]

So stated, however, the convention leads to strange, even paradoxical, consequences.[3] Consider, for example, N. Høeg's **A**:

---

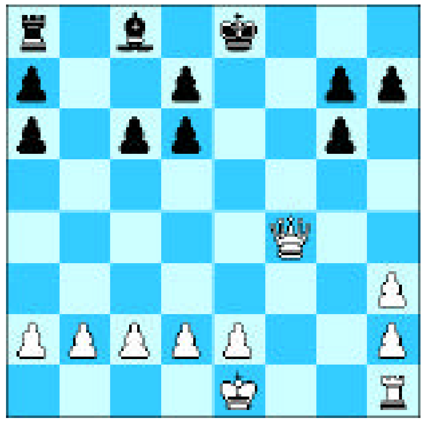
[1]To take one example more or less at random, the CCC is endorsed as one of several "universally accepted problem stipulations" by Michael Lipton, R. C. O. Matthews and John M. Rice in *Chess Problems: Introduction to an Art*, Faber & Faber, 1963, p. 19.

[2]See A. S. M. Dickins, *A Guide to Fairy Chess*, Dover, 1971, p. 39. The formulation given in Article 16, paragraph 1 of the *Codex for Chess Composition* issued by the Permanent Commission of the FIDE for Chess Compositions is this: "Castling is deemed to be permissible unless it can be proved that it is not permissible." (From http://www.sci.fi/~stniekat/pccc/codex.htm.)

[3]One of the first to argue this was J. G. Mauldon in the 1960s in private exchanges with K. Fabel, reported in the latter's *Introduction to Retrograde Analysis*, Philip Cohen (trans.), The Q Press, 1983 (published in French in *Problème*, No. 74, March 1971). See Fabel's section 11, "Controversial Questions," for an interesting discussion. Fabel's own view is not entirely clear to me and, while sympathetic to some consequences of the one defended here, he seems to diverge from it. See below, for further discussion.

A

Niels Høeg, *Die Schwalbe*, July 1923



White to mate in 3

If we apply the CCC across the board, we conclude that White can O–O and also that Black can O–O–O. But if White can castle, then the wQ on f4 is promoted and retroanalysis reveals that Black cannot castle (the promoting wP from f2 must have disturbed the bK). We arrive at a contradiction.

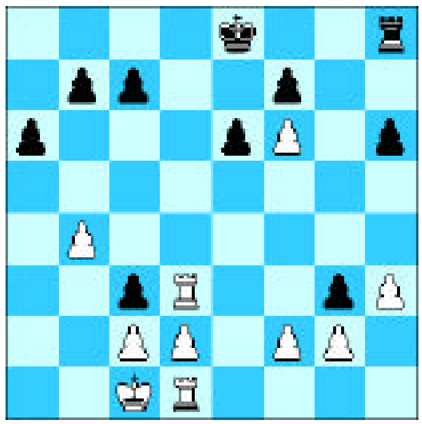
The CCC allows us to infer from "It cannot be shown that *X* is not legal" to "*X* is legal." This leads to problems because it might be that, while neither *X* nor *Y* can be shown not to be legal, *X* and *Y* cannot both be legal. Obviously, if the CCC is not to generate contradictions of this kind, it must be used to determine the castling possibilities of one side, and then, *given this determination*, be used to infer to the castling possibilities of the other side.

The composer's intention here in fact illustrates this use of the CCC. His solution is that White moves 1.Q×P, and now Black's only escape from 2.Rf1 is O–O–O. But after 1....Bb7, White preemptively moves 2.O–O (permitted by the CCC), and now Black cannot castle. Dickins says of this problem: "Their rights to castle are therefore mutually exclusive, and in such a case it is the player with the move who has the prior right, in this case White, whose turn it is to play."[4] The Piran Codex concurs with this elaboration of the CCC, and declares that "In the case of mutually exclusive White and Black castling, the one with the move has the prior right."[5]

The following two problems are suggestive in this connection.

B

R. Kofman, *Shakhmaty Bulletin*, 1958 (version)



White retracts last move and then mates in 3

In the diagram, if Black can O–O, then no mate in two exists (1.d2×c3 fails to 1....O–O). However, if White's last move was O–O–O, then retraction yields a position in which Black cannot O–O. (If White can O–O, then the wK has not moved; but if the wK has not moved, then the wR on d3 must have promoted; and in doing so, it must have disturbed either the bK or bR.) Thus after this retraction, White can mate in three by 1.O–O–O!

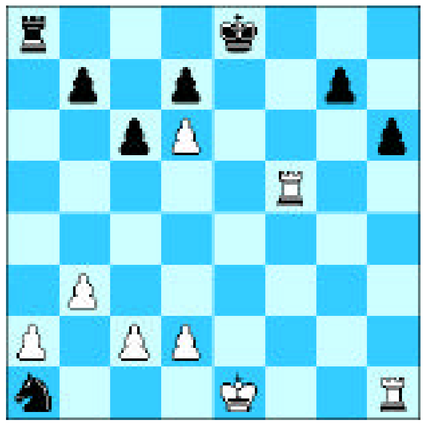
[4] *A Guide to Fairy Chess*, p. 24. Though perhaps this comment is rather an articulation of a position with which Dickins disagrees (though this is not obvious from the context). For later, he seems to take it back in Addendum No. 3, p. 64, where he appears to cite approvingly the view that problems "that claim to *prove* some retro-analytical fact by the *act* of castling are regarded as having 'No Solution' by experts such as T.R.D., L. Ceriani and Dr. Fabel—since the mere act of castling does not *prove* that it is a legal act." See also his endorsement of Dawson's "golden words" quoted below.

[5] See *A Guide to Fairy Chess*, p. 39. Article 16, paragraph 2 of the FIDE's PCCC's *Codex for Chess Composition* states: "In case of mutual dependency of castling rights of each party, the party exercising this right first is entitled to do so." (From http://www.sci.fi/~stniekat/pccc/codex.htm.)

Consider now the following closely related direct–mate:

C

Alexander George, original (after Kofman)



Mate in 2

According to the Piran Codex, this problem is a sound mate in two, solved by 1.O–O (followed by 2.Rf8); this is legal, because we apply the CCC first to White. Black cannot O–O–O in reply because its rook or king must have moved to allow promotion to the wR on f5—for this rook could not have originated on a1, otherwise White would not be able to castle.

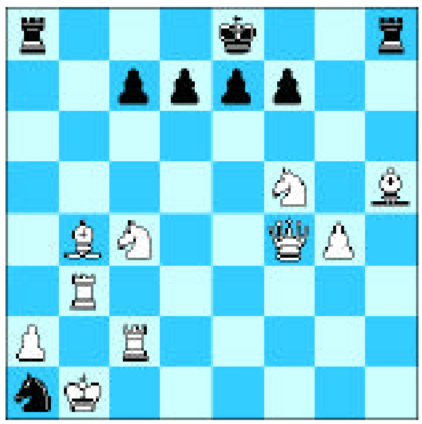
The CCC needs to be handled carefully. If it is applied baldly, it yields a contradiction here, as before: for we can show that White and Black cannot both castle in **C**. And if applied to Black first, then there is no mate; for we would then conclude that Black can O–O–O, and hence that White cannot O–O. (Note that 1.Rf1 fails to 1....S×c2+.)

But while careful handling of the CCC avoids conflicts between White and Black castling, these examples kindle a different dissatisfaction. For there is a natural way of understanding the problem **C** poses according to which the CCC, even if hedged to avoid such contradictions, is obfuscatory. We can view **C** as challenging us to determine whether *all* possible histories to the diagram position are such that White can mate in two moves. And far from helping us answer *this* question, the CCC actually obscures the matter by ruling out of court relevant possible histories. In this case, for instance, the CCC would have us not consider the situation in which Black has retained castling rights; and in these circumstances, there is no way for White to mate in two. The same holds for **A**: if the position has arisen in a game in which Black can O–O–O—as it surely could—then White cannot mate in three. Reliance on the CCC summarily blocks consideration of retroanalytical possibilities that may be relevant to a complete solution to these problems.

Before pursuing this, let us return to the prospect of contradiction. Consider:

D

J. G. Mauldon, *The Problemist*, January 1967



(a) Mate in 3
(b) a2 → b2. Mate in 2

Here, we cannot establish that Black cannot O–O; so, by the CCC, we must conclude that Black can O–O. Similarly, we cannot show that Black cannot O–O–O; therefore, by the CCC, Black can O–O–O. Therefore, in (b) Black can O–O and also O–O–O. But we can also show that in (b) Black cannot both O–O and O–O–O (otherwise Black would have no last move). We have a contradiction.

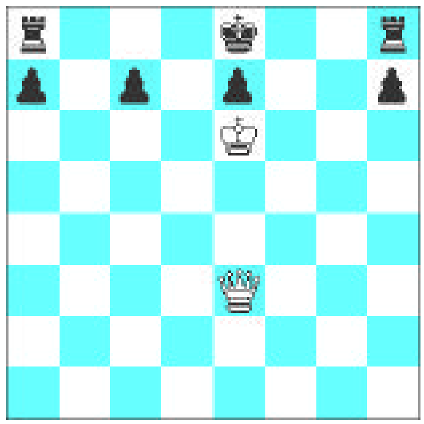
This does not reflect unsoundness in the problem: if Black can O–O, then White has 1.Qe5; if Black can O–O–O, then White plays 1.Qe4. (In part (a), it cannot be ruled out that Black can castle both sides, since Black's last move may have been b2×a1=S; so

1.Qe5? O–O–O! and 1.Qe4? O–O!. Rather, 1.Qe3!.) The contradiction stems not from the problem, but from a particular convention about castling.

D's ancestry can be traced back to Sam Loyd's famous two–mover:

E

Sam Loyd, *Texas Siftings*, 1888 (?)



Mate in 3

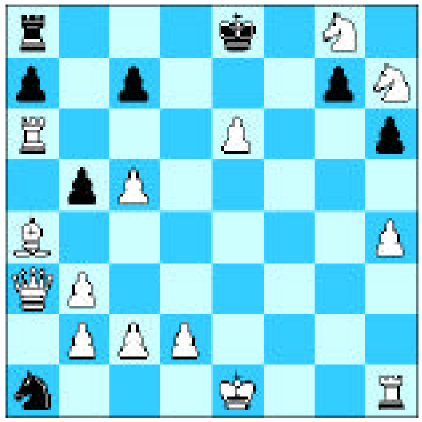
Its solution depends on whether Black can O–O–O (1.Qd4!) or O–O (1.Qg5!), where it can be established that Black cannot do both. The CCC, while it again leads to contradiction in reasoning about the history of this position, is again irrelevant to the soundness of the problem: if Black cannot castle at all, then either keymove will do.

The last two problems show that even if some contradictory consequences of the CCC are avoided by applying the convention serially, as the Piran Codex suggests, not all are. In particular, conflicts involving same-side castling remain.

Consideration of L. Ceriani's **F** raises yet further issues:

F

L. Ceriani, *Europe Echecs*, 1960



Mate in 2

Here, application of the restricted CCC leads us to infer that White, who has the move, can O–O. It follows that Black cannot O–O–O: for if White can O–O, then the wR on a6 is promoted and the promotion must have caused the bK to have moved (as no wP could have made enough captures to promote on b8 or c8). White mates with 1.O–O, followed by 2.Rf8.

But all is not well, for this misses the other half of the composer's intention, which is that if Black has the right to O–O–O, then b7–b5 must have been Black's last move, allowing White to play 1.P×P *e.p.*, followed by 2.Qf8. This part of the problem cannot be so much as considered if we adhere to the CCC as modified in the Codex. And, as before, use of the unmodified CCC leads straight to a contradiction.

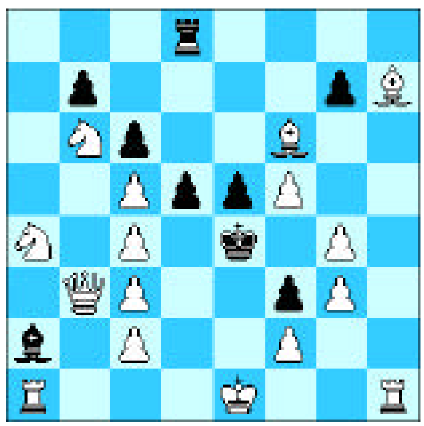
There is also a different kind of conflict lurking just under the surface, for the CCC clashes here with the convention for *e.p.* capture. If we apply the latter to the diagram position, then, because we cannot prove that Black's last move was b7–b5, we will conclude that White *cannot* capture *en passant*. Yet it is also true that we cannot prove that Black cannot castle, so by the CCC (assume we are told that Black has the move in the diagram position), we would conclude that Black can castle. But if Black can castle then Black's last move must have been b7–b5, and we now conclude that White *can* capture *en passant*. Here we have a contradiction, not between different applications of the CCC, but between application of the CCC and the reasonable convention governing *e.p.* capture.

Finally, as before, the CCC leads to an artificial narrowing of the retroanalytical possibilities considered. Thus it might well be that the position in **F** arose as a result of a sequence of moves that permit neither side to castle. Yet no mate is provided for this case: both 1.Pc6 and 1.Rf1 fail to 1....S×c2+.[6]

Let us now consider T. R. Dawson's famous **G**:

G

T. R. Dawson, *Retrograde Analysis*, 1915



Mate in 2

If we apply the CCC to White, as the Piran Codex dictates, we are led straight into contradiction. We cannot prove that White cannot play O–O (the wPs have made six captures, including a promoted piece, but the a7 bP could have promoted on a1). Nor can we establish that White cannot play O–O–O (since the h7 bP could have promoted on h1). Hence, by the CCC, White

[6]Those in the grip of the CCC would disagree with this analysis. Thus John Nunn, commenting on this problem, writes: "In fact, the solution depends on which player gains the benefit of the castling convention first. However, in each case there is a unique solution, so this problem is perfectly legitimate." (*Solving in Style*, George Allen & Unwin, 1985, p. 172.) And A. S. M. Dickins holds that **F** provides "a full solution for all retro-analytical possibilities." ("Alice in Retro-Land (or, Leap Before You Look)," *The Problemist*, September 1973, pp. 375-6; p. 376.)

can castle either side. But we can prove that this cannot be the case: for without a black promotion on a1 or h1 there would be too few black pieces to be captured by the wPs.

Dawson's reasoning was this: if Black last moved d7–d5 or e7–e5, then either the bB on c1 or the bB on f1 was not captured by the wPs, and both Black rook pawns must promote to be captured by wPs. This prohibits White from castling, but White can then play 1.c5×d5 *e.p.*, 1.f5×e5 *e.p.*, respectively. If Black's last move was other than d7–d5 or e7–e5, then we cannot show that White is unable to castle; therefore, we infer (by the CCC) that White can castle on one side or the other, which leads to mate in two.

Dickins finds this problem acceptable because it has "*four* solutions to meet *four* contingencies, the mates are all separated and there is neither cook nor dual."[7] This is correct, as far as it goes. But it is clear that there are more than four contingencies, a fact obscured by reliance on the CCC: it is possible that the diagram position was reached by a sequence of moves which leaves White unable to castle either side and which terminated in Black's d7–d6; if this were the case, White cannot mate in two. If one takes the stipulation "Mate in two" to mean that however the diagram position has arisen White can mate Black in two moves, then this problem has no solution. Ironically, Dickins follows his comment by saying that the "golden words" of Dawson "need repeating: —'Problems which prove partially some retrograde fact cannot be held to prove it absolutely ... it is necessary to state the alternatives'."[8] But Dawson's **G**, relying as it does on the CCC, is precisely an example of a problem that proves only partially some "retrograde fact," namely, that however the diagram position was reached, White can mate in two.

There are two main lessons suggested by these examples. First, the CCC is not a convention that can always be applied without generating contradiction. The convention can not

---

[7]*A Guide to Fairy Chess*, p. 64.

[8]Ibid. Dickins does not reference the quotation from Dawson. In the *Fairy Chess Review,* Dawson, apparently addressing himself to Ceriani, says this:

> Problems which prove only partially some retrograde fact cannot be held to prove the fact absolutely. Given that IF White may OOO, then Black may NOT play OOO—from retro reasoning—and vice versa, does not give evidence that White may start 1.OOO and so prevent OOO in reply. It is necessary to state the alternatives. I think your contention is correct. (February 1950, p. 95.)

only come into conflict with itself, but it can also come into conflict with the relatively unproblematic convention governing *e.p.* capture. Second, to apply the CCC to determine castling possibilities is always to rule out of consideration possible histories of the position which would deprive some side of this ability and which might be relevant to determining whether the stipulated goal of the problem can be attained. These are both serious deficiencies. But if we decide not to rely on the CCC, how shall we proceed instead?

The approach of *partial retrograde analysis* (PRA) has been developed to cope with these difficulties.[9] The general idea is that because retroanalysis cannot always determine the relevant history of the diagram position, the solution to a problem might have to divide into a number of cases corresponding to the relevantly different possible histories of the position. This much seems on the right track. Challenges arise, however, in spelling out the proposal.

Before considering some suggestions, a terminological point. "*Partial* retrograde analysis" is an unfortunate name for an approach that in fact aims to remedy partiality through a complete analysis of all possible historical cases. I shall, therefore, adopt the name "complete retroanalysis by cases" (CRAC) for the approach to be developed. Now let us return to more substantive issues.

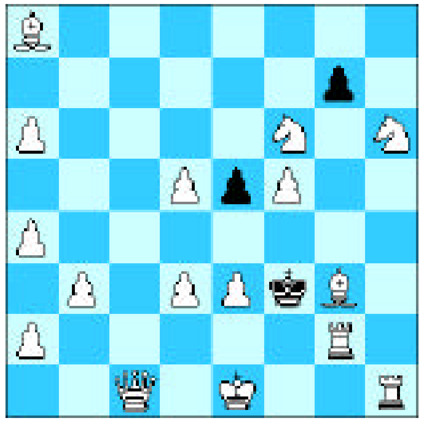
Dickins once suggested that a problem be divided into cases corresponding to *possible last Black moves*.[10] The difficulty with this is that it inevitably forces one's analysis to rely again on the CCC! For examining possible last Black moves will often not suffice to determine castling possibilities, which might be sensitive, say, to whether a promotion took place several moves earlier; and in such circumstances, recourse will inevitably be made to the CCC. This can be seen by considering K. Fabel's **H**:

[9] The approach is also known as *retro-variants* (RV).

[10] A. S. M. Dickins, *Die Schwalbe*, March-April 1969.

H

K. Fabel, *The Problemist*, 1969



White to mate in 1

The composer's reasoning was this: Black's last move was either e7–e5 or e6–e5. In the first case, White mates with 1.d5×e5 *en passant.* (1.O–O is not legal because, given the wP captures and the bB otherwise captured on f8, a promotion must have taken place on h1.) In the second case, since we cannot establish that White cannot castle, White can play 1.O–O, *by the CCC.* The reasoning is similar to that in Dawson's **G**. It gives the illusion of offering an exhaustive analysis of the possible histories to the diagram position, but in fact it fails to because within some of these cases, the CCC is relied upon. Thus, in **H** Black's last move might have been e6–e5 *and* White might be unable to castle. We should reject a conception of complete retroanalysis that allots any role to the CCC (and one that restricts itself to scrutiny of possible last moves must); otherwise, we shall make complete retroanalysis blind to those histories that are arbitrarily ruled out by the CCC.

How then are we to construe CRAC so as to accomplish this? Another suggestion is to treat retroanalytical problems as "twins."[11] Thus a problem like **F** would come in two parts: (a) White can castle, and (b) Black can castle. This does not seem satisfactory, however. Consider,

[11]Fabel credits Ceriani with first advancing the idea in his *La Genesi delle Posizioni*. A somewhat different version is explored by Gerd Rinder in "'Partielle Analyse' oder 'Retro-Strategie'?," *Die Schwalbe*, April 1970, pp. 95-99.

for instance, the fate of problem **H** on this proposal: if we are thoroughly to free ourselves from the CCC, it seems we would have to say that **H** is a twin with stipulation "Mate in 1: (a) Black last moved e7–e5; (b) Black last moved e6–e5 and White can castle."[12] This seems quite awkward. These suggested stipulations are as unmotivated as "White to mate in 2 and Black cannot play *c*," where *c* is a defense to which White has no response. Normally, the device of twinning is justified by the fact that the solutions in the twins exhibit some thematic relation to one another. In this case, by contrast, the device's effect is to decree the legality of moves that the composer failed to force through the positioning of pieces. Even in those cases in which the twinning device does not lead to awkwardness, it is less than desirable for it effectively does part of the solving work for the solver; part of a problem's challenge might well be the discovery of all relevant possible histories.[13]

In determining how to proceed, it will be of value to return to the question of castling and to think through what our attitude towards it should be.

In an orthodox direct-mate problem, White is to mate Black against any defense. It is natural, then, to construe White's task in a retroanalysis direct-mate to be this: given any sequence of legal moves that leads to the position, mate Black against any defense. If a possible

[12]Rinder appears to conclude that **H** is not in fact a problem in more than one part. Its solution, according to him, is simply 1.O-O. His reasoning (p. 98) seems to be this: he claims that if we assume that White can castle, then e7-e5 was not the last move, and that if we assume that e7-e5 was not the last move, then White can castle. Even were these conditional claims true, we could not infer that White can castle. At any rate, the second is false (unless we appeal to the CCC): it is possible that e7-e5 was the last move and that White cannot castle. Rinder's method seems to stem from his recognition that the application of common retroanalytic conventions yields different results depending on the order in which they are applied. His response is that we should determine how many parts a problem has by applying these conventions *in all possible orders* and seeing how many distinct outcomes result (*Prinzip der permutierten Fragen*). But as already noted, the CCC is also objectionable because it always obscures relevant possible histories of the diagram position. Thus, even if one does consider all possible permutations, one's results will be incomplete if one relies on the CCC. For instance, in applying Rinder's method to various problems, W. Keym never considers—precisely because he relies on the CCC—the possibility that neither side can castle. (W. Keym, "Konstruktive Kritik am Kodex von Piran," *Die Schwalbe*, October 1972, pp. 389-94; pp. 392-3.)

[13]Fabel, in his *Introduction to Retrograde Analysis*, seems to endorse the construal of PRA problems as twins. He cites approvingly (p. 11) what he takes to be Ceriani's (and others') position: "PRA problems are considered 'twin' problems in which, to be sure, it is not always sufficient to write the symbol 'PRA' under the diagram. In 'critical cases', ... it is necessary to supplement this with rulings on the possibility of, *e.g.*, castling and *en passant* capture." For the reasons just given, I do not find this a satisfactory outcome.

route to the diagram position allows Black the defense of castling, then White must be able to answer it. Of course, some routes to the position will deprive Black of that ability, and White must be able to mate in those circumstances as well. We should not assume that the keymove in those cases in which Black can castle must also provide the solution, or even be legal, in all those situations in which Black cannot castle. Ceriani's **F** illustrates this: if it is assumed that Black can castle, then the keymove is White's *e.p.* capture, a move that may be unavailable to White in the absence of this assumption. Consequently, we cannot make any one assumption about Black's options. *White must be able to mate in the case that Black can castle (unless, of course, it can be proved that Black cannot) and also in the case that Black cannot castle, if necessary by specifying different keymoves depending on the history of the game that led to the diagram position.*

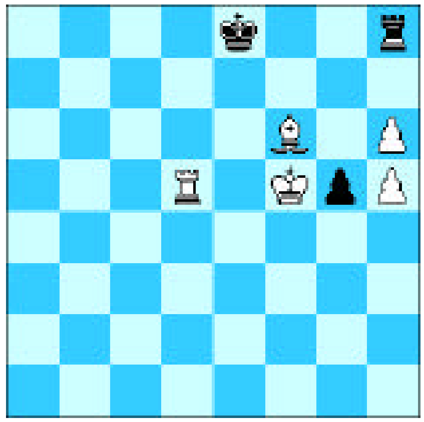
The potential multiplicity of keymoves is not plausibly viewed as a violation of the demand for uniqueness of solution: for this requirement is not that there be exactly one keymove that brings about mate in the stipulated number of moves whichever sequence of legal moves yields the diagram position. The scope of the quantifiers, as logicians would say, must be reversed: rather, the demand is at most that for any sequence of legal moves that yields the diagram position there should be exactly one keymove that will mate in the stipulated number of moves. Both problems **D**, part (b), and **E** are problems whose keymoves vary with the history to the diagram position we are considering; in both cases, White's keymove depends on what we assume about Black's castling abilities. This is not plausibly taken as a violation of the standard constraint on uniqueness of solutions.

But what does the uniqueness constraint actually require? Let us distinguish between two different constraints. *Strong uniqueness* demands that for each keymove *k*, if *h* is a history of the diagram position that has *k* as its keymove then no other keymove will work for *h*. *Weak uniqueness* requires that for each keymove *k,* there be at least one history of the diagram position that has *k* as its only keymove.

To get a sense of the consequences of our choice, consider W. Langstaff's **I**:

I

W. Langstaff, *The Chess Amateur*, 1922



White to mate in 2

The composer's reasoning is clearly this: Either Black can O–O or Black cannot. If Black can, then Black's last move was g7–g5 and White can play 1.h5×g5*e.p.*, followed by either 2.Rd8 or 2.h7. If Black cannot castle, then we cannot establish that the bP moved last; but in this case, 1.Ke6 leads to mate.

This problem satisfies weak uniqueness: 1.h5×g5*e.p.* is the only keymove that mates if Black can castle, and 1.Ke6 is the only keymove that works if Black cannot castle. But **I** does not satisfy strong uniqueness: in the history of the position in which Black cannot castle and Black's last move was g7–g5, either keymove will mate in two. Likewise, problems **D**, part (b), and **E** fail strong uniqueness.

A problem that satisfies strong uniqueness is akin to one without any duals. A problem that satisfies weak, but not strong, uniqueness is instead analogous to one that contains duals, but only minor ones. Since minor duals are usually tolerated, we shall adopt weak uniqueness as our requirement governing uniqueness of solutions in CRAC problems.[14]

[14]Rinder objects (p. 97) to Mauldon's proposal, essentially the position we shall come to, on the ground that it fails strong uniqueness. He suggests that this is tantamount to countenancing duals. Though Rinder appears to treat all duals as equal, the concept of separation permits one to distinguish between major duals and minor ones, the latter usually being tolerated. By extension, weak uniqueness should be too. At any rate, for the benefit of

We saw that White must be prepared to respond to Black's castling (if indeed there is a possible history in which castling is permitted). Can White's keymove ever be castling? Assume that there is a possible history to the diagram position, call it h, that would allow White to castle. Could the corresponding keymove, call it k_h, ever be castling? We know that there is a different possible history to the diagram position, h', which is identical to h except for a few otherwise irrelevant moves that make White's castling impossible. (For instance, let h' be identical to h except for the following moves prefixed to h: 1.Sc3 Sc6, 2.Rb1 Sb8, 3.Ra1 Sc6, 4.Sb1 Sb8, 5.Sf3 Sc6, 6.Rg1 Sb8, 7.Rh1 Sc6, 8.Sg1 Sb8.) Since h' does not permit White to castle, if the problem is not to be without a solution White must be able to mate through a keymove other than castling, call it $k_{h'}$. But now observe that $k_{h'}$ must—in addition to k_h—be a keymove for the position that arose through h. This situation violates strong uniqueness because corresponding to history h there will be two keymoves, namely k_h and $k_{h'}$. But more importantly, weak uniqueness is also violated, for there is no history for which k_h is the sole keymove. It is not true of any castling keymove that there is a possible history to the diagram position that has only it for a keymove; for any history whose keymove is castling, there will be another keymove that will turn the trick for White as well. *In sum, any CRAC direct-mate problem that allows White castling as a keymove is unsound*.[15]

It does not follow that problems of the form "Can White castle?" are unsound or uninteresting. The only two justifiable answers to such a question are "No" and "Perhaps."

purists, I note below how the general conception here advanced can be reformulated so as to impose the requirement of strong uniqueness.

[15]Rinder (p. 97) believes that this introduces an unjustified asymmetry between Black and White. It is no more unjustified, however, than the fact that, while Black can make any number of responses to White, White can have at most one response to any Black move (with the exception of minor duals). This asymmetry is simply a consequence of the fact that in a direct-mate chess problem, there must be a unique way in which White can mate Black against any defense. The asymmetry is constitutive of the different roles occupied by White and Black in direct-mate problems.

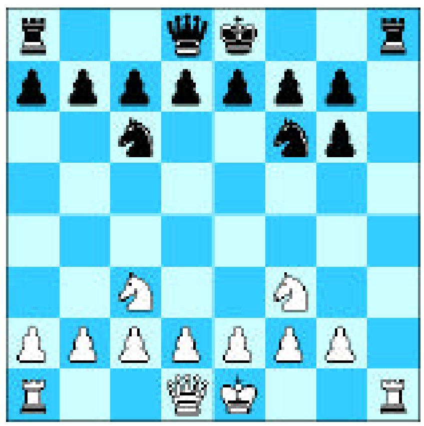
It is interesting that our conclusions, while at odds with the framework adopted by many modern composers, harks back to earlier attitudes. Thus Brian Harley advised composers:

> to avoid publication of problems where *White* Castling would affect the real solution, unless his admission of the manoeuvre is clearly understood by all his readers. In the case of a possibility of *Black* Castling as a defence, it would be as well to provide a successful reply, whether the composer believes in allowing the manoeuvre or not. (*Mate in Two Moves*, G. Bell & Sons, London, 1931, p. 9.)

Setting a problem that calls upon the solver to determine which of these is the correct answer is a perfectly legitimate goal. The CCC converts these answers into "No" and "Yes," respectively. Nothing is gained by this, while consistency and accuracy may be sacrificed. If one wishes to pose a yes/no question, then one may simply ask instead "Might White be able to castle?" For example, problem **J**, which originally appeared with the question "Which side can castle?," can easily be reformulated as follows:

J

Alexander George, *The Problemist*, November 1983 (version)



Which side might be able to O–O?

(a) Diagram
(b) Remove Ra1
(c) Remove Ra8
(d) Remove both Ra1 and Ra8

A general view can now be made explicit as follows. We shall say that an orthodox direct-mate is a sound *n–fold CRAC problem in m moves* if its solution consists of n moves k_1, k_2, …, k_n such that (1) for each history of the position, there is at least one k_i that will lead to mate in m moves; (2) for each k_i, there is at least one history of the position for which k_i leads to mate in m moves; and (3) for each k_i, there is at least one history of the position for which no k_j different from k_i leads to mate in m moves.[16]

[16]See also B. P. Barnes' "Twins" in *The Problemist*, March 1967, p. 121.

In this analysis, (1) insures that the problem has a solution; (2) guarantees that the problem is indeed an ***n***-fold CRAC; while (3) insures that the problem is not cooked because of incomplete separation.[17] The analysis of course agrees with the conclusions we reached above concerning castling: (1) entails that all possible histories to the diagram position be considered, including those in which Black is able to castle (unless it can be shown that there are none) and those in which Black is not able to castle; and it is a consequence of (3) that castling cannot be a keymove.[18]

For example, **D**, part (b), is a 2–fold CRAC in three moves. Each history to the diagram position is such that either 1.Qe4 or 1.Qe5 mates in three moves; and 1.Qe4 is the only keymove that leads to mate in three if the history is such that Black can O–O–O, while 1.Qe5 is the only keymove that leads to mate in three if the history is such that White can O–O.

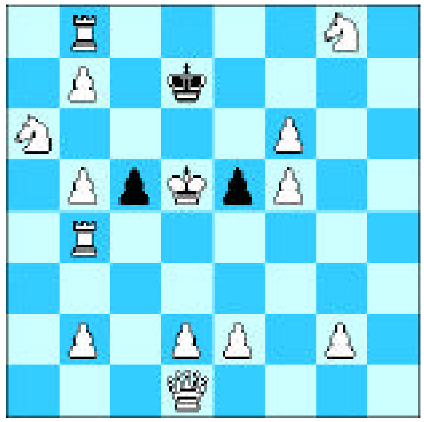
In light of this analysis, we can now see that even the standard convention for *en passant* pawn capture is not completely accurate as it stands. Consider **K**:

[17]Strong uniqueness can be enforced by strengthening (1) to: (1') for each history of the position, there is exactly one k_i that will lead to mate in m moves. (3) can now be dropped as superfluous, as it is a consequence of (1') and (2). Since (1) is a consequence of (1'), it follows, as desired, that any problem satisfying strong uniqueness must also satisfy weak uniqueness. Because (1) and (2) and (3) taken together fail to entail (1'), a problem could satisfy weak uniqueness without satisfying strong uniqueness, again the desired result.

[18]Rinder might object (p. 97) that such a proposal would have us declare many great works unsound. It is true that many would not be sound CRAC problems. But even though CRAC is the most natural and rigorous interpretation of a direct-mate retroanalytical problem, composers are free to formulate unusual stipulations; perhaps the CCC might be viewed as a particular form of fairy retroanalysis. It is also worth remembering that an unsound composition might well retain its historical, constructional, and even aesthetic significance; many (what we now consider) flawed problems of the great pioneers of the nineteenth century have done so.

K

Alexander George, original



White to mate in 1

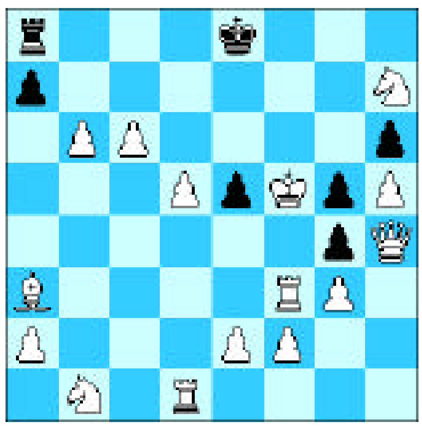
One can prove neither that Black's last move was c7–c5, nor that it was e7–e5. Therefore, assuming the standard convention for *e.p.* capture, White can play neither 1.b5×c5 *e.p.*, nor 1.f5×e5 *e.p.*, and the problem is without solution. From the present perspective, however, no special convention is needed: the problem is simply a sound 2–fold CRAC problem in one move. And this is the desired conclusion, since clearly Black's last move was a double move by some pawn and mate will follow. Interestingly, while reliance on the CCC often obscures the unsoundness of problems, the standard convention for *e.p.* capture can conceal a problem's soundness.

Loyd's **E** illustrates one mechanism for achieving a 2–fold CRAC: a position that could be legally reached with Black having retained the right to O–O or the right to O–O–O, but not both, each situation calling for a different White keymove. **K** illustrates a second mechanism: a position in which Black's last move must have been one of two double pawn moves, each demanding a different White response. The third mechanism for attaining a 2–fold CRAC involves both castling and *en passant* capture: a position in which, either Black has lost the right to castle on one side or Black's last move was a double pawn move, where White's keymove will depend on which of the two possible histories gave rise to the position. Langstaff's **I** is a simple example of such a problem.[19]

There are likewise three mechanisms whereby a 3–fold CRAC can be achieved. In the first, different keymoves will be required depending on whether (i) Black's last move was a double move of pawn P_1 and Black has retained the ability to castle; (ii) Black's last move was a double move of pawn P_2 and Black has retained the ability to castle; and (iii) Black's last move was not a double move of either P_1 or P_2 and Black has lost the ability to castle. **L** illustrates this mechanism:

L

Alexander George, *The Problemist*, January 1992



White to mate in 2

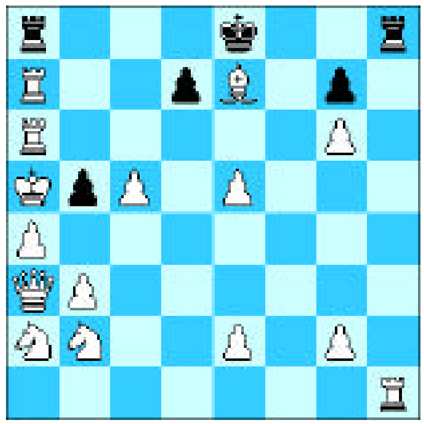
The second mechanism requires a position in which different keymoves are called for depending on whether the diagram position was reached in such a way that (i) Black's last move was a double move of pawn P, and Black retains the ability both to O–O and to O–O–O; (ii) Black's last move was not a double move of pawn P, and Black retains the ability to O–O but not to

[19]The descriptions of these and the following mechanisms could of course be refined. For example, the last subsumes both those problems in which Black has lost the right to castle kingside or Black's last move was a double pawn move, and those in which Black has lost the right to castle queenside or Black's last move was a double pawn move. It is easy enough to work out an exhaustive classification from the mechanisms presented here.

O–O–O; and (iii) Black's last move was not a double move of pawn P, and Black retains the ability to O–O–O but not to O–O. **M** provides an example of this mechanism:

M

Alexander George, *The Problemist*, January 1992

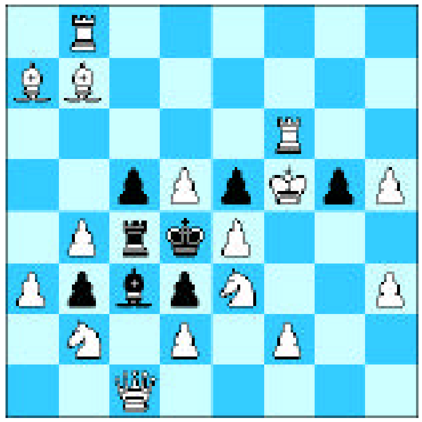


White to mate in 2

The third mechanism that permits a 3–fold CRAC is a position in which one can establish that Black's last move was one of three double pawn moves. Dawson's ingenious and original **N** illustrates this:

N

T. R. Dawson, *Retrograde Analysis*, 1915



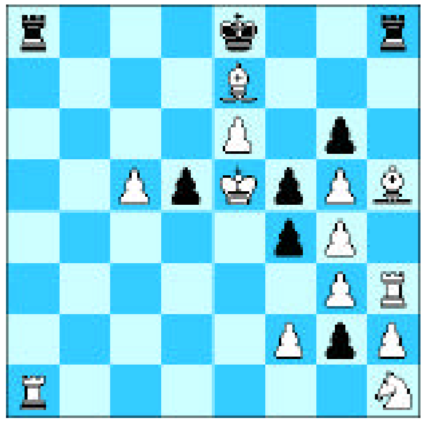
White to mate in 2

This is the maximum number of double pawn moves that can be forced.

There are two theoretically distinct ways of achieving a 4–fold CRAC. First, through a position in which different White keymoves are called for depending on whether (i) Black retains the ability to castle, the last move being a double move of pawn P_1; (ii) Black retains the ability to castle, the last move being a double move of pawn P_2; (iii) Black retains the ability to castle, the last move being a double move of pawn P_3; (iv) Black's last move was not a double move of either P_1, P_2, or P_3, and Black no longer has the right to castle. The second mechanism is used in a position in which (i) Black retains the right to O–O and to O–O–O, the last move being a double move by pawn P_1; (ii) Black retains the right to O–O and to O–O–O, the last move being a double move by pawn P_2; (iii) Black's last move was not a double move by either P_1 or P_2, and Black retains the right to O–O only; (iv) Black's last move was not a double move by either P_1 or P_2, and Black retains the right to O–O–O only. Both W. Keym's **O** and J. G. Mauldon's **P** illustrate this second mechanism.

O

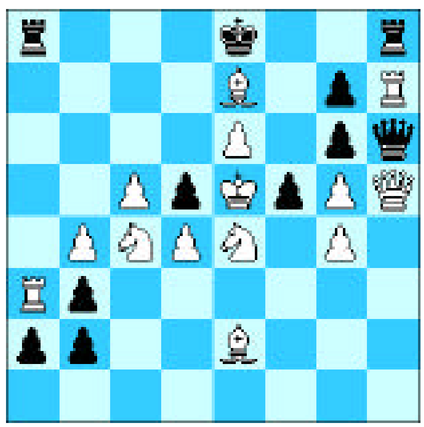
W. Keym, *Die Schwalbe*, 1971



White to mate in 3

P

J. G. Mauldon, *British Chess Magazine*, December 1965



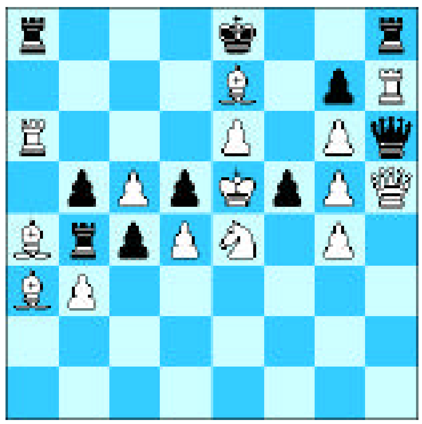
Can White mate in 3?

(a) Diagram
(b) White pawn on g6

Finally, there is only one mechanism for achieving a 5–fold CRAC, the theoretical maximum. The position must be one that requires White's keymoves to vary depending on whether the history of the diagram position was such that (i) Black retains the right to O–O and to O–O–O, the last move being a double move by pawn P_1; (ii) Black retains the right to O–O and to O–O–O, the last move being a double move by pawn P_2; (iii) Black retains the right to O–O and to O–O–O, the last move being a double move by pawn P_3; (iv) Black's last move was not a double move by P_1 or P_2 or P_3, and Black retains the right to O–O only; (v) Black's last move was not a double move by P_1 or P_2 or P_3, and Black retains the right to O–O–O only. Mauldon achieved this in 1967:

Q

J. G. Mauldon, *British Chess Magazine*, June 1966 (version)



White to mate in 3

Finally, any orthodox direct-mate that, as it were, 'does not involve retroanalysis' is of course simply a 1–fold CRAC problem.

Appendix on the Constructivist Retroanalyst

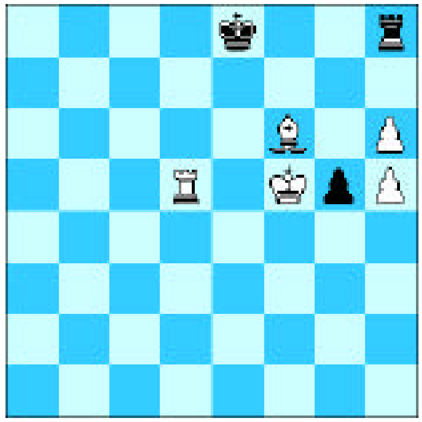
Constructivism in logic or mathematics issues from the perspective that refuses to allow a general appeal to the determinate nature of mathematical reality in the course of proving assertions. Rather, a particular proof, or construction, must accompany each claim, the precise nature of which will vary with the logical structure of the claim being established.

For instance, in order to prove a disjunction "*X* or *Y*," the constructivist demands nothing less than a proof of *X* or a proof of *Y*. The non-constructive logician – sometimes called the "classical" logician – makes no such demand; he might be willing to accept an assertion of that form even though he is neither in a position to prove *X* nor prepared to prove *Y*.

Consider the particular disjunction "*X* or not-*X*." A classical logician will accept every such disjunction, whatever *X* might be; indeed, this is often known as the *Law of the Excluded Middle*. This is because such a logician imagines that, for any meaningful statement *X*, the mathematical universe is such that either *X* is true or *X* is false. The constructivist, by contrast, makes no such assumption: if one wishes to assert "*X* or not-*X*" then one must either present a proof of *X* or present a proof of not-*X*. It might well be that one is neither in a position to prove *X* nor in a position to prove not-*X*; for the constructivist, there would then be nothing for it but to refrain from asserting the disjunction "*X* or not-*X*."

Any n-fold CRAC (for $n > 1$) problem would be acceptable to a "classical problemist." Consider for instance Langstaff's **I**, repeated here:

W. Langstaff, *The Chess Amateur*, 1922



White to mate in 2

Either the history of the game is such that Black can castle (viz., neither its king nor its rook has moved) or it is such that Black cannot castle. If Black can castle, then Black's last move must have been Pg7-g5 (for Black has moved neither king nor rook, and Pg6-g5 would have placed White in an impossible checked position). But in that case, White can move 1.Ph5×Pg5 *e.p.*, leading to mate the following move (either 2.Rd8 or, if Black castles, Ph7). On the other hand, if Black cannot castle, then White can move 1.Ke6 (but not 1.Ph5×Pg5 *e.p.* since we cannot now show that Black's last move must have been Pg7-g5), which leads to mate (namely, 2.Rd8). Summarizing: either White can capture *e.p.* or Black cannot castle; if the first, then 1.Ph5×Pg5*e.p.* mates; if the second, then 1.Ke6 mates; therefore, either 1.Ph5×Pg5*e.p.* mates or 1.Ke6 mates – even though there is no way for us to know which of these two moves is the actual keymove.

The "constructivist problemist" is willing to agree that *if* White can *e.p.* capture *then* 1.Ph5×Pg5*e.p.* mates, and also that *if* Black cannot castle *then* 1.Ke6 mates. To this degree, he can sign on to the "classical" analysis. But he cannot go the extra step and conclude, as his classical counterpart does, that either 1.Ph5×Pg5*e.p.* mates or 1.Ke6 mates. This is because the constructive retroanalyst will not assert the disjunction "White can capture *e.p.* or Black cannot castle." He will not assert this because he cannot establish either of the two disjuncts: he cannot show that White can capture *e.p.*, nor can he show that Black cannot castle.

One might wish to object to the constructivist that surely we *can* establish that in Langstaff's problem it cannot both be the case that White cannot capture *e.p.* and Black can castle. And the constructivist will agree: the assumption that Black can castle forces the conclusion that Black's last move was a double pawn move which permits White to capture *en passant*. Thus the assumption that both White cannot capture *e.p.* and Black can castle leads to a contradiction; and that indeed justifies our claim that it cannot be that both conditions hold. But this claim – which is of the form "not-(not-*X* and *Y*)," where *X* is "White can capture *e.p.*" and *Y* is "Black can castle" – does not force the constructive retroanalyst to agree that either White can capture *e.p.* or Black cannot castle – which is a claim of the different form "*X* or not-*Y*." For this inference – from "¬(ϕ   ψ)" to "¬ϕ   ¬ψ" – is precisely one whose general validity is disputed by the constructivist: to have shown that one cannot establish both ϕ and ψ is not have shown that ϕ cannot be established nor to have shown that ψ cannot be established.[20]

Likewise, one might try to urge upon the constructivist retroanalyst that surely *if* Black can castle *then* White can capture *en passant*. And furthermore, "if *Y* then *X*" is logically equivalent to "not-*Y* or *X*"! Does this not force the constructivist to accept the disjunction? No again. It is true that the constructivist will agree with the conditional claim: for any proof that Black can castle can easily be transformed into a proof that White can capture *en passant*. But the sticking point is that for the constructivist "ϕ → ψ" does not logically entail "¬ϕ   ψ": for an ability to transform any proof of ϕ into a proof of ψ might not give one the means either to prove ¬ϕ or to prove ψ.[21]

Thus, while the constructivist retroanalyst can agree to many claims his classical counterpart makes with regard to the logic of Langstaff's problem, he will not take the final step and state the solution disjunctively; he will not baldly assert "either 1.Ph5×Pg5*e.p.* mates or 1.Ke6 mates."

---

[20] The particular reasoning here also involves moving from "not-not-ϕ" to "ϕ," which is yet another inference disputed by the constructivists.

[21] For more information about constructivism – its philosophical basis and the logic and mathematics attending it — the reader may wish to consult Alexander George and Daniel J. Velleman, *Philosophies of Mathematics*, Blackwell, 2002; see especially Chapter 4, "Intuitionism."

More generally, while a classical retroanalyst should have no qualms about the solution of an n-fold CRAC problem – in particular, with the claim that either k_1 mates or k_2 mates or … or k_n mates –his constructive counterpart, while able to follow him quite a distance, will have scruples of a philosophical nature regarding this final disjunctive formulation of the solution.[22]

[22] I do not believe that the idea of using retroanalysis problems to illustrate the difference between classical and constructive reasoning is original to me, but all my efforts to locate a source have failed.

Abbreviated Solutions to Selected Problems

J: (a) Neither side can castle; (b) only Black might be able to castle; (c) only White might be able to castle; (d) both Black and White might be able to castle.

L: Depending on whether Black has just moved e7–e5, g7–g5, or Black cannot castle, White moves 1.d5×e5 *e.p.*, 1. h5×g5 *e.p.*, or 1.K×g4.

M: Depending on whether Black has just moved b7–b5, Black cannot castle kingside, or Black cannot castle queenside, White moves 1.c5×b5 *e.p.*, 1.Bg5, or 1.Bd6.

N: Depending on whether Black has just moved c7–c5, e7–e5, or g7–g5, White moves 1.d5×c5 *e.p.*, 1.d5×e5 *e.p.*, or 1.h5×g5 *en passant.*

O: Depending on whether Black has just moved d7–d5, f7–f5, Black cannot castle kingside, or Black cannot castle queenside, White moves 1.c5×d5 *e.p.*, 1.g5×f5 *e.p.*, 1.Bf6, or 1.Bd6.

P: (a) No guaranteed mate as Black may be able to castle either side. In (b), mate can be forced but the keymove depends on whether Black's last move was d7–d5, f7–f5, or whether Black cannot castle queenside, or whether Black cannot castle kingside; the corresponding keymoves are 1.c5×d5 *e.p.*, 1.g5×f5 *e.p.*, 1.Bd6, or 1.Bf6.

Q: Depending on whether Black has just moved b7–b5, d7–d5, f7–f5, or whether Black cannot castle kingside, or Black cannot castle queenside, White moves 1.c5×b5 *e.p.*, 1.c5×d5 *e.p.*, 1.g5×f5 *e.p.*, 1.Bf6, or 1.Bd6.

ACKNOWLEDGMENTS

An earlier version of this essay was completed in June 1992 and published in *diagrammes* [France], July-September 1994, pp. 25-34. I owe thanks to J. G. Mauldon for many stimulating exchanges on these topics, to Susan George for valuable help with translations, and to Jill Cirasella for her creative graphic design.